УДК 539.12.01

LOW-ENERGY THEOREM IN SOFTLY BROKEN SUPERSYMMETRY

S.G.Kovalenko

We prove a general low-energy theorem recovering a generic relation between the neutrino Majorana mass and the superpartner sneutrino B–L-violating «Majorana»-like mass term. The theorem states, if one of these two quantities is nonzero, another one is also nonzero and, vice versa, if one of them vanishes, another vanishes, too. The theorem is a consequence of the underlying supersymmetry (SUSY) and valid for any model of weak scale softly broken SUSY.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

Низкоэнергетическая теорема в мягконарушенной суперсимметрии

С.Г.Коваленко

На теоремном уровне установлена низкоэнергетическая связь между нейтринным майорановским массовым членом и B–L-нарушающим майораноподобным массовым членом снейтрино — суперпартнера нейтрино. Теорема утверждает, что если одна из этих двух величин ненулевая, то отлична от нуля и другая, и наоборот, если одна из них строго нулевая, то должна зануляться и другая. Эта теорема является общим следствием суперсимметрии (SUSY) и справедлива для любой модели с мягким нарушением SUSY на фермиевском масштабе.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

Neutrinos are believed to be massive particles. Despite the lack of unambiguous experimental confirmation of this belief, there are insisting indications for nonzero neutrino masses from cosmology, the solar and atmospheric neutrino puzzles (for recent review see [1]) as well as from recent LSND results on possible $\bar{\nu}_e - \bar{\nu}_\mu$ neutrino oscillations [2].

Among the known explanations for the extreme smallness of the neutrino mass compared to masses of the other fermions the most natural one is based on the see-saw mechanism [3]. It leads to a B–L-violating Majorana mass term for the neutrino. Various 1-loop contributions to the neutrino self-energy, widely discussed in the literature [4]-[9], also induce a small Majorana mass for neutrinos. Furthermore, the Grand Unification paradigm definitely prefers a Majorana mass for neutrinos. Due to these arguments, it has become a common trend to think of neutrinos as Majorana particles.

In supersymmetric (SUSY) models the neutrino ν has its scalar superpartner — the sneutrino $\tilde{\nu}$. Given that they are components of the same superfield, one may suspect a

certain interplay between the neutrino and sneutrino properties at low energies as a relic of the underlying supersymmetry.

In the present note we prove a low-energy theorem establishing an intimate relation between the neutrino Majorana mass term and the *B–L*-violating as well as *B–L*-conserving sneutrino mass terms. Our consideration refers to the general structure of the low-energy effective Lagrangian assuming weak scale softly broken supersymmetry and stability of the ground state after electro-weak symmetry breaking. Proof of the low-energy theorem is based on symmetry arguments and has a general significance.

The effective Lagrangian of a generic model of weak scale supersymmetry contains after electro-weak symmetry breaking the following terms

$$\mathcal{L} = -\sqrt{2}g\varepsilon_i \cdot \bar{\nu}_L \chi_i \tilde{\nu}_L - g\varepsilon_i^- \cdot \bar{e}_L \chi_i^- \tilde{\nu}_L - g\varepsilon_i^+ \cdot \bar{\nu}_L \chi_i^+ \tilde{e}_L + \tag{1}$$

$$+ \frac{g}{\sqrt{2}} \cdot \bar{\nu}_L \gamma^\mu e_L W_\mu^+ + g \cdot \bar{\chi}_i \gamma^\mu (O_{ij}^L P_L + O_{ij}^R P_R) \chi_j^+ W_\mu^- + \ldots + h.c.$$

Dots denote other terms which are not essential for further consideration. Here, $\tilde{\nu}_L$ and $\tilde{e}_L$ represent scalar superpartners of the left-handed neutrino ν_L and electron e_L fields. The chargino χ_i^+ and neutralino χ_i are superpositions of the gaugino and the higgsino fields. Contents of these superpositions depends on the model. Note that the neutralino is a Majorana field $\chi_i^c = \chi_i$. The explicit form of the coefficients ε, $\varepsilon_i^\pm$ and $O_{ij}^{L,R}$ is also unessential. For the case of the MSSM one can find them, for instance in [10]. Eq.(1) is a general consequence of the underlying weak scale supersymmetry (softly broken) and the spontaneously broken electro-weak gauge symmetry.

Now let us assume the neutrino acquires a Majorana mass

$$\mathcal{L}_M^\nu = -\frac{1}{2} m_M^\nu \bar{\nu}^c \nu, \tag{2}$$

where $\nu = \nu_L + \nu_L^c$ is a Majorana field. The further proof does not depend on the mechanism generating this mass term in the low-energy Lagrangian. For the sake of simplicity and without any loss of generality we ignore possible intergeneration neutrino mixing.

Let us prove the following statements.

Statement 1: If $m_M^\nu \neq 0$, then in low-energy Lagrangian also the sneutrino «Majorana» like *B–L*-violating mass term is present

$$\mathcal{L}_M^{\tilde{\nu}} = -\frac{1}{2} \tilde{m}_M^2 \tilde{\nu}_L \tilde{\nu}_L - h.c. \tag{3}$$

with $\tilde{m}_M^2 \neq 0$. Note that $\tilde{m}_M^2$ is not a positively defined parameter.

Statement 2 is an inverse to the Statement 1: If in Eq.(3) $\tilde{m}_M^2 \neq 0$, then in Eq.(2) $m_M^\nu \neq 0$.

First notice that in the presence of a nonzero Majorana neutrino mass term in Eq.(2) the «Majorana»-like sneutrino mass term in Eq.(3) is generated at the 1-loop level as shown in Fig.1(a) with neutrino and neutralino internal lines. An opposite statement based on the

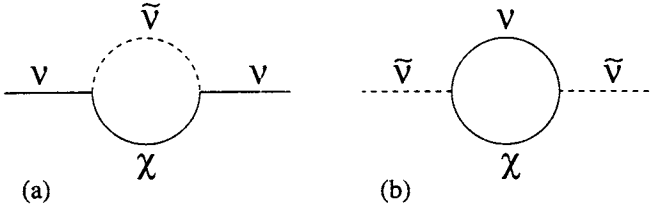

Fig.1. One-loop contributions (a) to the neutrino Majorana mass m_M and (b) to the sneutrino B–L-violating mass $\tilde{m}^2_M$

1-loop diagram in Fig.1(b) is also true. The B–L-violating sneutrino propagator in Fig.1(b) is proportional to $\tilde{m}^2_M$ and explicitly derived below. Here we do not need detailed calculations and write down schematically

$$\tilde{m}^2_M = \frac{1}{4\pi}(g^2 + g'^2) m^{\nu}_M M_{\chi i} \gamma_i + A_{\tilde{\nu}}, \qquad (4)$$

$$m^{\nu}_M = \frac{1}{4\pi}(g^2 + g'^2) \frac{\tilde{m}^2_M}{\tilde{m}^2_D} M_{\chi i} \beta_i + A_{\nu}. \qquad (5)$$

Here m_D is a mass parameter explained below, g and g' are the $SU(2)_L \times U_{1Y}$ gauge coupling constants, $M_{\chi i}$ are the neutralino masses, and β_i, γ_i are functions depending on neutralino mixing coefficients and on the masses of particles in the loop. $A_{\tilde{\nu}}$ and A_{ν} represent any other possible contributions. The explicit form of A_i, β_i, γ_i is not essential for us. What is important is just the presence of a correlation between $\tilde{m}^2_M$ and m^{ν}_M which we write down in general as $\tilde{m}^2_M = f(m^{\nu}_M)$, $m^{\nu}_M = \phi(\tilde{m}^2_M)$. Now we are going to prove that

$$\tilde{m}^2_M = f(m^{\nu}_M = 0) = 0, \quad m^{\nu}_M = \phi(\tilde{m}^2_M = 0) = 0. \qquad (6)$$

One can expect such properties of the functions f and ϕ from Eqs.(4)-(5). Indeed, $\tilde{m}^2_M = 0$ in the left-hand side of Eq.(4) strongly disfavors $m^{\nu}_M \neq 0$. Similarly, $m^{\nu}_M = 0$ in the left-hand side of Eq.(5) strongly disfavors $\tilde{m}^2_M \neq 0$. This is because vanishing of left-hand sides of Eqs.(4)-(5) requires either vanishing of both terms in the right-hand sides or their net cancellation. The latter is unlikely since it implies unnatural fine-tuning of certain parameters. More serious, it is unstable under radiative corrections. Even if the fine-tuning was done by hand, it would be spoiled in higher orders of perturbation theory. To guarantee the cancellation of both terms in the right-hand sides of Eqs.(4)-(5) in all orders of perturbation theory one needs a special unbroken symmetry. The Lagrangian (1) does not possess any continuous symmetry having nontrivial B–L-transformation properties. How-

ever, there might be an appropriate discrete symmetry. Let us specify this discrete symmetry group by the following field transformations

$$\nu \to \eta_\nu \nu, \quad \tilde{\nu} \to \eta_\nu \tilde{\nu}, \quad e_L \to \eta_e e_L, \quad \tilde{e}_L \to \eta_{\tilde{e}} \tilde{e}_L, \tag{7}$$

$$W^+ \to \eta_W W^+, \quad \chi_i \to \eta_{\chi_i} \chi_i, \quad \chi^+ \to \eta_{\chi^+} \chi^+.$$

Here η_i are phase factors. Since the Lagrangian (1) is assumed to be invariant under these transformations one obtains the following relations

$$\eta_\nu^* \eta_{\tilde{\nu}} \eta_{\chi_i} = 1, \quad \eta_e \eta_{\chi^+} \eta_{\tilde{\nu}}^* = 1, \tag{8}$$

$$\eta_e \eta_W \eta_\nu^* = 1, \quad \eta_W^* \eta_{\chi^+} \eta_{\chi_i}^* = 1.$$

This defines the admissible discrete symmetry group of the Lagrangian in Eq.(1).

Solving these equations, one finds

$$\eta_\nu^2 = \eta_{\tilde{\nu}}^2. \tag{9}$$

This relation proves the statements 1, 2 and the corresponding properties expressed by Eqs.(6). To see this we note that the *B–L*-violating mass terms in Eq.(2) or in Eq.(3) are forbidden by this symmetry if $\eta_\nu^2 \neq 1$ or $\eta_{\tilde{\nu}}^2 \neq 1$. Contrary, if $\eta_\nu^2 = 1$, this mass term is not protected by the symmetry and appears in higher orders of perturbation theory, even if it does not exist at the tree-level. Relation (9) claims that if the neutrino Majorana mass term in Eq.(2) is forbidden $m_M^\nu = 0$, then the sneutrino Majorana-like mass term in Eq.(3) is also forbidden $\tilde{m}_M = 0$ and vice versa. If one of them is not forbidden, they are not forbidden both. Thus, Statements 1,2 and explicit relations in Eqs.(6) are proven.

One can derive the following corollary from statements 1,2.

Corollary: If one of the two *B–L*-violating masses, either m_M^ν or $\tilde{m}_M^2$, vanishes, then another one vanishes too.

Let us turn to the last Statement.

Statement 3: In the presence of $\tilde{m}_M^2 \neq 0$ in Eq.(3) there must exist a «Dirac»-like *B–L*-conserving sneutrino mass term

$$\mathcal{L}_D^{\tilde{\nu}} = -\tilde{m}_D^2 \tilde{\nu}_L^* \tilde{\nu}_L \tag{10}$$

with $\tilde{m}_D^2 \geq |\tilde{m}_M^2|$.

To prove this statement consider the combined sneutrino mass term $\mathcal{L}_{\text{mass}}^{\tilde{\nu}} = \mathcal{L}_M^{\tilde{\nu}} + \mathcal{L}_D^{\tilde{\nu}}$ and use the real field representation for the complex scalar sneutrino field $\tilde{\nu} = (\tilde{\nu}_1 + i\tilde{\nu}_2)/\sqrt{2}$, where $\tilde{\nu}_{1,2}$ are real fields. Then

$$\mathcal{L}_{\text{mass}}^{\tilde{\nu}} = -\frac{1}{2} \tilde{m}_1^2 \tilde{\nu}_1^2 - \frac{1}{2} \tilde{m}_2^2 \tilde{\nu}_2^2, \tag{11}$$

where $\tilde{m}^2_{1,2} = \tilde{m}^2_D \pm |\tilde{m}^2_M|$. Assume the vacuum state is stable. Then $\tilde{m}^2_{1,2} \geq 0$, otherwise subsequent spontaneous symmetry breaking occurs via $\langle \tilde{\nu}_i \rangle \neq 0$. Broken symmetry in this case is the *R*-parity. It is a discrete symmetry defined as $R_p = (-1)^{3B+L+2S}$, where *S*, *B* and *L* are the spin, the baryon and the lepton quantum number.

This completes the proof of the theorem consisting of the above three statements.

From the above consideration it follows that a self-consistent structure of mass terms of the neutrino-sneutrino sector is

$$\mathcal{L}^{\nu\tilde{\nu}}_{\text{mass}} = -\frac{1}{2} m^{\nu}_M \bar{\nu}^c \nu - \frac{1}{2} (\tilde{m}^2_M \tilde{\nu}_L \tilde{\nu}_L + h.c.) - \tilde{m}^2_D \tilde{\nu}^*_L \tilde{\nu}_L. \tag{12}$$

The Dirac neutrino mass term $m^{\nu}_D(\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L)$ can also be introduced but it is not required by the self-consistency arguments.

It is instructive to derive an explicit form of the above mentioned *B*–*L*-violating sneutrino propagator. It can be done by the use of the real field representation as in eq.(11). Present for comparison both the *B*–*L*-conserving $\mathcal{D}^{(+)}_{\tilde{\nu}}$ and the *B*–*L*-violating $\mathcal{D}^{(-)}_{\tilde{\nu}}$ sneutrino propagators

$$\mathcal{D}^{(+)}_{\tilde{\nu}}(y-x) = \langle 0 | T(\tilde{\nu}^*(x)\tilde{\nu}(y)) | 0 \rangle = \frac{1}{2} \langle 0 | T(\tilde{\nu}_1(x)\tilde{\nu}_1(y)) | 0 \rangle + \tag{13}$$

$$+ \frac{1}{2} \langle 0 | T(\tilde{\nu}_2(x)\tilde{\nu}_2(y)) | 0 \rangle = -\frac{i}{2} (D^c_{\tilde{m}_1} + D^c_{\tilde{m}_2}),$$

$$\mathcal{D}^{(-)}_{\tilde{\nu}}(y-x) = \langle 0 | T(\tilde{\nu}(x)\tilde{\nu}(y)) | 0 \rangle = \frac{1}{2} \langle 0 | T(\tilde{\nu}_1(x)\tilde{\nu}_1(y)) | 0 \rangle - \tag{14}$$

$$- \frac{1}{2} \langle 0 | T(\tilde{\nu}_2(x)\tilde{\nu}_2(y)) | 0 \rangle = -\frac{i}{2} (D^c_{\tilde{m}_1} - D^c_{\tilde{m}_2}),$$

where

$$D^c_{\tilde{m}_i}(x) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ikx}}{\tilde{m}^2_i - k^2 - i\varepsilon} \tag{15}$$

is the ordinary propagator for a scalar particle with mass m_i. Using the definition of $\tilde{m}_{1,2}$ as in Eq.(11) one finds

$$\mathcal{D}^{(+)}_{\tilde{\nu}}(y-x) = \int \frac{d^4k}{(2\pi)^4} \frac{\tilde{m}^2_D - k^2}{(\tilde{m}^2_1 - k^2 - i\varepsilon)(\tilde{m}^2_2 - k^2 - i\varepsilon)} e^{-ikx}, \tag{16}$$

$$\mathcal{D}^{(-)}_{\tilde{\nu}}(y-x) = -\tilde{m}^2_D \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ikx}}{(\tilde{m}^2_1 - k^2 - i\varepsilon)(\tilde{m}^2_2 - k^2 - i\varepsilon)}. \tag{17}$$

It is seen that in the absence of the *B*–*L*-violating sneutrino Majorana-like mass term $\tilde{m}^2_M = 0$, the *B*–*L*-violating propagator vanishes while the *B*–*L*-conserving one becomes the ordinary propagator of a scalar particle with mass $\tilde{m}_1 = \tilde{m}_2 = \tilde{m}_D$.

In the presence of the *B*–*L*-violating sneutrino Majorana-like mass term the complex scalar sneutrino field splits into two real mass eigenstate fields $\tilde{\nu}_{1,2}$ with different masses $\tilde{m}_{1,2}$. The square mass splitting is $2\tilde{m}^2_M$.

This sneutrino mass splitting parameter can be proved by searching for B–L-violating exotic processes such as neutrinoless double beta decay. It is obvious from Eqs.(4)-(5) that certain constraints on $\tilde{m}^2_M$ can be also obtained from the experimental upper bound on the neutrino mass. We are going to analyse these constraints in a separate paper.

In summary, we have proven a low-energy theorem for weak scale softly broken supersymmetry relating the B–L-violating mass terms of the neutrino and the sneutrino.

Acknowledgments

We thank V.A.Bednyakov for helpful discussions. The research described in this publication was made possible in part by Grant CNTR 215NUCLON from the Russian Ministry of Science.

References

1. Smirnov A.Yu. — Plenary talk given at 28th International Conference on High Energy Physics, 25—31 July 1996, Warsaw, Poland, hep-ph/9611465;
   Suzuki Y. — Plenary talk at the same Conference.
2. The LSND Collaboration, C.Athanassopoulos et al. — Phys. Rev. Lett., 1996, v.77, p.3082; ibid. 1995, v.75, p.2650; nucl-ex/9605001.
3. Gell-Mann M., Ramond P., Slansky R. — In: Supergravity, ed. F. van Nieuwenhuizen and D.Freedman, North Holland, Amsterdam, 1979, p.315;
   Yanagida T. — Proc. of the Workshop on Unified Theory and Baryon Number of the Universe, KEK, Japan, 1979;
   Weinberg S. — Phys. Rev. Lett., 1979, v.43, p.1566.
4. Lee I.H. — Nucl. Phys., 1984, v.B246, p.120; Phys. Lett., 1984, v.B138, p.121.
5. Mohapatra R., Nussinov S., Valle J.W.F. — Phys. Lett., 1985, v.B165, p.417.
6. Barbieri R. et al. — Phys. Lett., 1990, v.B238, p.86.
7. Hall L., Suzuki M. — Nucl. Phys., 1984, v.B231, p.419.
8. Ma E., Hg D. — Phys. Rev., 1990, v.D41, p.1005.
9. Zee A. — Phys. Lett., 1980, v.B93, p.389.
10. Haber H.E., Kane G.L. — Phys. Rep., 1985, v.117, p.75;
    Nilles H.P. — Phys. Rep.., 1984, v.110, p.1.

Received on December 16, 1996.